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INVESTIGATION OF THE CONNECTION BETWEEN SOIL AND GROUND WATERS WITH IRRIGATION

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With a close occurrence of the level of the ground waters, capillary influx from below can be a significant source of replenishment of the moisture reserves of the root-inhabited layer. The value of this influx depends on the depth of the occurrence of the level of the ground waters, the water and physical properties of the soils of the aeration zone, the form of agricultural cultivations, and the meteorological conditions.

Determination of the rate of the capillary influx is required, in the first place, for calculation of irrigation norms and the irrigation curve and, in the second place, to find the optimal depth of the occurrence of the ground waters, with the aim of preventing processes of secondary salinization, occurring in the case of mineralized ground waters and saline soil waters of the aeration zone [1].

§1. We consider one-dimensional not-fully-established filtration in a vertical direction in a thickness of soil (taking account of its inhomogeneous lithological makeup) from the surface of the ground to the level of the ground waters.

From the solution of the differential equation describing the motion of the water in the unsaturated zone

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left[k(\theta,z)\left(\frac{\partial\psi}{\partial z}-1\right)\right]-f(\theta,z,t) \tag{1.1}$$

with the initial and boundary conditions

$$\psi(z,\ 0)=\psi^0(z),\ 0\leqslant z\leqslant l,\ t=0; \tag{1.2}$$

$$-k(\partial\psi/\partial z-1)=R(t),\ z=0,\ t>0; \tag{1.3}$$

$$\psi=0,\ z=l,\ t\geqslant 0 \tag{1.4}$$

and conditions of conjugation in the form of the equality of the pressures at the interface between the layers, a determination is made of the pressure $\psi(z, t)$ and, consequently, of the moisture content $\theta(z, t)$, if, for each lithological layer of the soil thickness under investigation, we know the main hydrophysical characteristics $\theta(\psi)$ and $k(\theta)$ [or $k(\psi)$], which are here assumed to be single-valued.

The function $f(\theta, z, t)$ in Eq. (1.1) takes account of the absorption of moisture by the roots of plants in the region $0 \le z \le z_r(t)$, where $z_r(t)$ is the thickness of the root zone. For $z > z_r$, we assume that $f \equiv 0$.

The rate of capillary influx from the ground waters into the aeration zone v is found from the balance equation

$$v(t)=\frac{dw}{dt}-R+\int_0^{z_r} f\,dz, \tag{1.5}$$

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where $w = \int_0^l \theta(z, t)\, dz$ is the moisture content of the investigated thickness.

The following formula is taken [2] as the dependence of the coefficient of moisture conductivity k on the moisture content θ:

$$k = k_s\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^n, \quad \theta_r < \theta \leqslant \theta_s, \quad k = 0, \quad \theta \leqslant \theta_r. \tag{1.6}$$

The dependence of the moisture content θ on the suction pressure ψ is given in the form [3]

$$\theta = \theta_0 + \frac{\theta_s - \theta_0}{1 + a(-\psi)^m}, \tag{1.7}$$

where k_s and θ_s are, respectively, the filtration coefficient and the moisture content with total saturation; θ_r, θ_0, a, m, and n are parameters characterizing types of soils ($\theta_r > \theta_0$) (θ_r is the least moisture content of the layer; ψ_r is its corresponding pressure).

Using the results of [4, 5], the function $f(\theta, z, t)$ can be represented in the form

$$f(\theta, z, t) = \varepsilon(t)\, p(\theta)\, q(z) \Big/ \int_0^{z_r(t)} p(\theta)\, q(z)\, dz, \tag{1.8}$$

where

$$q(z) = cz_r(t) - z; \tag{1.9}$$

$$p(\theta) = \begin{cases} 1, & \theta \geqslant \theta_c, \\ \dfrac{\theta - \theta_w}{\theta_c - \theta_w}, & \theta_w \leqslant \theta < \theta_c \quad (\theta_c \leqslant \theta_r), \end{cases} \tag{1.10}$$

θ_s is the critical moisture content; θ_W is the wilting moisture content. The quantity $\varepsilon(t)$ takes account of the equal potential transpiration, since it is postulated that the moisture content in the root zone lies within optimal limits.

The problem (1.1)-(1.4) is approximated to the second order on a grid nonuniform with respect to t, in accordance with the Crank–Nicholson scheme, and, on a grid uniform with respect to z, by a homogeneous difference scheme with discontinuities at the mesh points [6]. The system of difference equations obtained, nonlinear with respect to ψ, is solved by the Newton iteration method. Here the starting iteration is given by the extrapolation of values from the two preceding layers with respect to the time. The iteration process is ended when the maximal increment of the pressure with respect to z after the iteration becomes less than the previously assigned value, which is such that the error due to breaking-off the iteration process will be less than the error of the difference approximation.

Figures 1-4 show the rates of capillary influx v, obtained as the result of a numerical solution of the problem (1.1)-(1.10).

In all the variants, the initial profile of the pressure is assumed to be equilibrium:

$$\psi^0(z) = \begin{cases} \psi_r, & z \leqslant l + \psi_r, \\ z - l, & z > l + \psi_r \quad (0 \leqslant z \leqslant l). \end{cases}$$

For a real soil, type, and phase of development of agricultural cultivation, there exists an optimal interval of the moisture content of the root-inhabited layer, in which the rate of transpiration does not depend on the moisture content. The upper limit of this interval, as a rule, is the least moisture content θ_r, and the lower limit is the moisture content $\theta_c = \gamma\theta_r$, where $0.5 < \gamma \leq 1$.

To maintain the moisture content in the root zone within optimal limits, irrigation of the soil is assigned by the norm $(1 - \gamma)\langle \theta_r \rangle z_r(t)$ as soon as it falls to a value of $\langle \theta_c \rangle z_r(t)$. The duration of the irrigation is taken equal to 0.5 day, and the rate of irrigation $R = (1 - \gamma)\langle \theta_r \rangle z_r(t)/0.5$ m/day. Here $\langle \theta_r \rangle$ and $\langle \theta_c \rangle$ denote the mean values of θ_r and θ_c over the depth of the root zone. The start of irrigation on the curves is marked by a + sign, while the numbers on the curves denote the depth of the occurrence of the level of the ground waters l.

In all the variants, the following parameters, entering into formulas (1.6)-(1.10), had exactly the same values: m = n = 3, $\theta_s = 0.5$, $\theta_r = 0.32$, $\theta_0 = 0.15$, $\theta_c = 0.7$, $\theta_r = 0.224$, $\theta_w = 0.16$, c = 1.125.

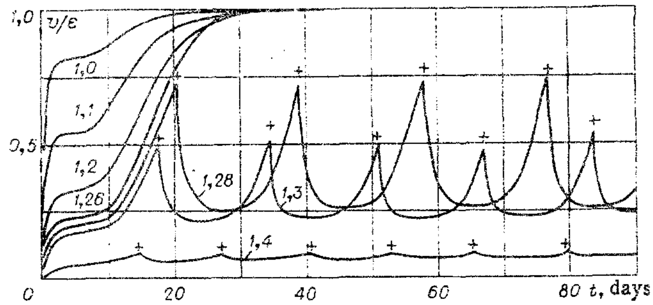

Fig. 1

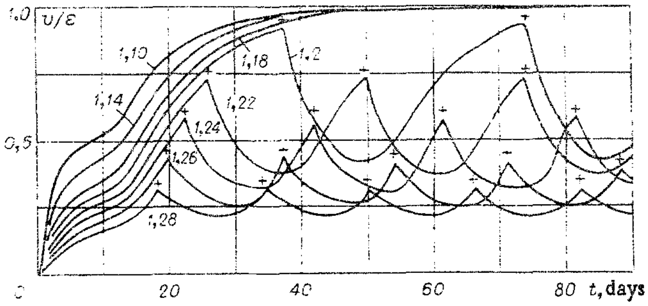

Fig. 2

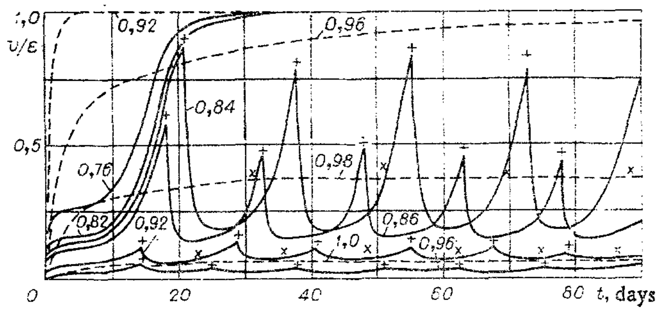

Fig. 3

The curves of v in Figs. 1-3 correspond to the case where the values of ε and z_r are constant for the course of the period under consideration and are equal to 0.004 m/day and 0.5 m.

In the first of the above variants (see Fig. 1), the investigated thickness of soil is assumed to be homogeneous with $k_s = 1$ m/day, $\psi_r = -1$ m, and $a = 1.06$ m^{-3}.

In the second variant (see Fig. 2) filtration in a two-layer soil with a filtration coefficient $k_s = 1$ m/day and a thickness of 0.5 m for the upper layer and $k_s = 0.1$ m/day for the lower layer was considered. The remaining parameters were the same as previously, i.e., the layers differed only in the values of the filtration coefficients.

Figure 3 shows curves for a two-layer soil with identical values of the filtration coefficients of the layers ($k_s = 1$ m/day), but having a different water-retaining capacity. The dashed lines correspond to the case where, above, there is a layer with a thickness of 0.5 m, having a large water-retaining capacity, with the parameter $a = 1.06$ m^{-3}, while, for the lower layer, $a = 8.48$ m^{-3}. Here $\psi_r = -1$ and -0.5 mm for the upper and lower layers, respectively. The solid lines correspond to the case where the upper layer has a lower water-retaining capacity.

An analysis of the curves of v shows that there exists a depth of the occurrence of the level of the ground waters l_*, which is such that, for $l \leq l_*$, the additional feeding of water is not required to maintain the moisture content in the root-inhabited layer within optimal limits, while, for $l > l_*$, an additional supply is required.

For the variants under consideration, $l_* = 1.26$ (see Fig. 1), 1.18 (see Fig. 2), and 0.96 and 0.82 m (see Fig. 3). It can be seen that a small increase in the depth of the occurrence of the level of the ground waters in comparison with l_* leads to a considerable decrease in the rate of capillary influx. Thanks to the constant values of ε and z_r, for depths $l > l_*$ there is a repetition of the behavior of v in the intervals between irrigations.

A comparison of the curves of v in Figs. 1 and 2 shows that, for exactly the same depths, in the case of a two-layer soil with a lower, less permeable layer, the value of the capillary influx is considerably less. If we change the places of the layers, v decreases even more strongly, although the character of the curves remains the same.

As can be seen from Fig. 3, the value and the character of the behavior of v depends to a considerable degree on the parameters characterizing the water-retaining capacity of the lithological layers.

Under real conditions, with the passage of time, there is a change in the values of the potential transpiration ε, depending on the meteorological conditions and on the type and stage of development of the plants;

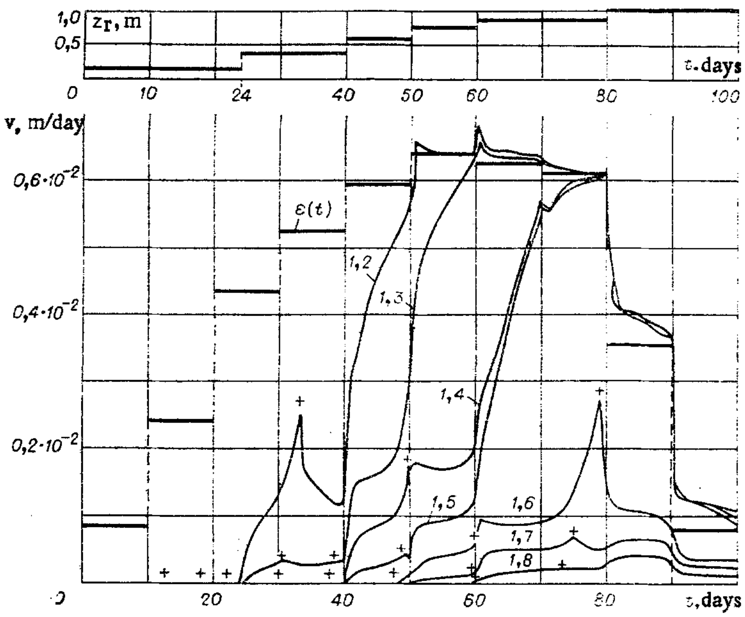

Fig. 4

TABLE 1

l, m	1,2	1,3	1,4	1,5	1,6	1,7	1,8
Relative norm, mm	77	113	171	171	330	330	330
Influx from ground waters, mm	306	254	180	164	44	20	8
Change in moisture reserve, mm	37	53	69	85	46	60	82

there is an increase in the thickness of the root zone. The curves of v in Fig. 4 were obtained taking account of the change of the values of z_r and ε with the time according to a power law. These data were taken for wheat from [7, 8], respectively. The remaining parameters were the same as in the first variant. The depth of the occurrence l of the level of the ground water varied from 1.2 to 1.8 m. Calculations show that, for depths $l >$ 1.6 m, the feeding of the aeration zone by ground waters practically ceases. For $z_r = 0.14$ m ($t \le 24$ days), when the capillary rim lies below the root zone, v = 0 for all values of l. With an increase in l, there is an increase in the period of time for which v = 0. Table 1 gives the distribution of the components of the total transpiration for a season, equal to 420 mm.

Figure 5, for $l = 0.9$, 1.0, 1.2 m, with the starting data for the first variant, gives the profiles of the moisture content which are established after the rate of the capillary influx v becomes practically equal to ε ($t > 35$ days), and, for $l = 1.3$ m, profiles of the moisture content corresponding to moments of time before the start of irrigation and after its end (dashed lines).

For determined depths of the occurrence of the level of the ground waters (LGW), a zone is formed, adjacent to the surface of the ground, in which the moisture content θ is equal to the wilting moisture content θ_w. Under these circumstances, the removal of moisture by the roots of plants takes place from deeper layers of the soil. For $l = l_*$, the thickness of this zone z_* is maximal, and the mean moisture content in the root zone $\langle\theta\rangle = \langle\theta_c\rangle$. An increase in l leads to a decrease in z_* and, starting from some depth, this zone vanishes altogether; the value of $\langle\theta\rangle$ for these cases becomes greater than $\langle\theta_c\rangle$. For depths for which additional irrigation is needed, the value of $\langle\theta\rangle$ in the root zone does not drop below $\langle\theta_c\rangle$, although, at individual points, the moisture content can be equal to the wilting moisture content.

The above-described method also makes it possible to investigate cases with a varying level of the ground waters, with an arbitrary distribution of the rate of precipitation, irrigation, and transpiration, and with the presence of hysteresis in the dependence of the pressure and the coefficient of moisture conductivity on the moisture content.

§2. The maximal depth of the occurrence of the level of the ground waters l_* for which irrigation is required can be found approximately from the solution of the following steady-state problem, using the known

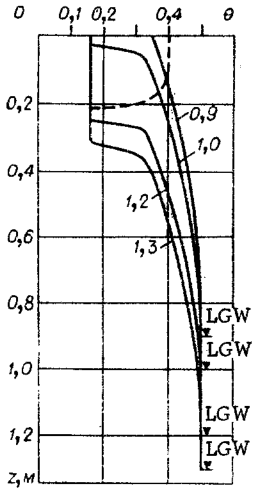

Fig. 5

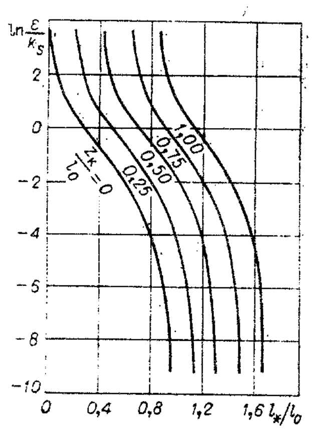

Fig. 6

values of ε and z_r:

$$\frac{d}{dz}\left[k(\theta, z)\left(\frac{d\psi}{dz}-1\right)\right]=f(z); \tag{2.1}$$

$$\psi=\psi_r(z_*),\quad k\left(\frac{d\psi}{dz}-1\right)=0,\quad z=z_*, \tag{2.2}$$

where $k(\theta)$ and $\theta(\psi)$ are determined from formulas (1.6) and (1.7) and f(z), from (1.8); it is assumed that $p(\theta) \equiv 1$, since $\theta \geq \theta_r \geq \theta_c$ and

$$q(z)=c(z_r-z_*)-(z-z_*). \tag{2.3}$$

The values of ε and $(z_r - z_*)$ are assumed to be constant. It is required to find the value of z for which $\psi = 0$.

The error in f, arising from the replacement of (1.9) by (2.3), is equal to zero for c = 1, and, for c > 1, does not exceed $f(z_r)(z_*/z_r)/(2c-1)$, where $f(z_r)$ is the least value of f(z).

The replacement $\xi = z - z_*$ makes it possible to eliminate the previously unknown quantity z_*. Integration of Eq. (2.1) with respect to ξ, taking account of (2.2) and of the condition of the equality of the flows at the discontinuities, gives

$$\frac{d\psi}{d\xi}=1+\left(\int\limits_0^{\xi} f(\zeta)\,d\zeta\right)\Big/k(\theta,\xi). \tag{2.4}$$

Expanding the indeterminacy in Eq. (2.4) for $\xi = 0$, we find

$$\left.\frac{d\xi}{d\psi}\right|_{\psi=\psi_r(0)}=\begin{cases}0, & n>1,\\ 1\Big/\left(\frac{1}{2}+\sqrt{\frac{1}{4}+f(0)\frac{1+\alpha}{m}}\right), & n=1,\\ 1, & n<1,\end{cases} \tag{2.5}$$

where

$$\alpha = a|\psi_r(0)|^m.$$

Equation (2.4) is brought into the form

$$\frac{d\xi}{d\psi}=k\Big/\left(k+\int\limits_0^{\xi} f(\zeta)\,d\zeta\right). \tag{2.6}$$

Since $k \neq 0$ inside the region, by virtue of the fact that the flow does not revert to zero, then, from the condition of the equality of the pressures at the discontinuities, it follows that the function $\xi(\psi)$ is continuous. This makes it possible to integrate (2.6) numerically by the Runge–Kutta method with respect to ψ from $\psi_r(0)$ to 0 with values of ξ and $d\xi/d\psi$ at the initial point $\psi = \psi_r(0)$ known from (2.2), (2.5). Thus, we find the values of the difference $(l_* - z_*)$ as a function of ε and $\xi_c = z_r - z_*$.

In addition, the value of $\xi(\psi)$ is obtained in each stage of the integration. Using these values and the dependence $\theta(\psi)$, we can calculate the mean moisture content $\bar{\theta}(\varepsilon, \xi_c)$ over the part of the root zone $z_* < z < z_r$. On the other hand, since the mean moisture content in the zone $z < z_*$ is equal to the mean wilting moisture content θ_w, then

$$\bar{\theta}(\varepsilon, z_r - z_*) = (\langle\theta_c\rangle - \theta_w)[z_r/(z_r - z_*)] + \theta_w. \quad (2.7)$$

With a given value of z_r, Eq. (2.7) is solved graphically and the value of z_* is found. By the same token, l_* is determined as a function of ε and z_r.

The results of a calculation of l_* for the case of a homogeneous soil with $m = n = 3$, $c = 1.125$ are shown in Fig. 6 by the dependences of $\ln(\varepsilon/k_s)$ on l_*/l_0 for z_r/l_0 (where $l_0 = |\psi_r|$) and $\alpha = al_0^m = (\theta_s - \theta_r)/(\theta_r - \theta_0) = 1.06$. As $\varepsilon/k_s \to \infty$, the value of $l_* \to z_r(\theta_s - \theta_c)/(\theta_c - \theta_w)$, i.e., it depends linearly on z_r. As $\varepsilon/k_s \to 0$, the dependence $l_*(z_r)$, for a certain range of change in α and z_r, is close to linear, as can be seen in Fig. 6. For the case illustrated in Fig. 1, $l_* = 1.26$ m, and, from Fig. 6, $l_* = 1.25$ m. The small difference obviously arises as a result of the replacement of (1.9) by (2.3), as well as of smoothing of the curve of the dependence of moisture content on z in the neighborhood of z_* with a difference approximation of the unsteady-state problem. For $c = 1$, the dependence $l_*(\varepsilon/k_s, z_r)$, obtained from a solution of the steady-state problem, can be used to evaluate the accuracy of a difference scheme for the unsteady-state problem.

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